$A = \begin{pmatrix} (J_\varphi(s)\cdot e_1)^T \\ \vdots \\ (J_\varphi(s)\, e_n)^T \\ N^T(p) \end{pmatrix}$, $|A|>0$, $B=\begin{pmatrix} (J_\psi(t)\, e_1)^T \\ \vdots \\ (J_\psi(t)\, e_n)^T \\ N^T(p) \end{pmatrix}$, $|B|>0$. But $J_\psi(t) = J_{\varphi\circ h}(t)\cdot J_h(t) = J_\varphi(s) J_h(t)$ as $\psi = \varphi\circ h$.

where $N(p)$ is the orientation of S.

$AB^T = \begin{pmatrix} J_\varphi^T(s) \\ N^T(p)\end{pmatrix} (J_\varphi(s) J_h(t),\ N(p)) = \begin{pmatrix} J_\varphi^T(s)\cdot J_\varphi(s)\cdot J_h(t) & 0 \\ 0 & 1\end{pmatrix}$ So $|A|\cdot|B| = |J_\varphi^T(s) J_\varphi(s)|\cdot|J_h(t)|$

As $J_\varphi^T J_\varphi$ is positive semi-definite, $|J_\varphi^T(s) J_\varphi(s)| \geq 0$, But $|A|,|B|>0$. So $|J_h(t)|>0$.

Since p is any point on W and ψ is bijective, so $|J_h(t)|>0$ for any $t\in\psi^{-1}(W)$

Thus $\psi|_{\psi^{-1}(W)} = \varphi\circ h|_{\psi^{-1}(W)}$ is reparametrization of $\varphi|_{\varphi^{-1}(W)}$

17.19 Denote $x = X(p) = (x_1, x_2, x_3)$, $y = Y(p) = (y_1, y_2, y_3)$

$(W_X\wedge W_Y)(v,w) = W_X(v) W_Y(w) - W_X(w) W_Y(v) = (x\cdot v)\cdot(y\cdot w) - (x\cdot w)(y\cdot v)$

$= \left(\sum_{i=1}^3 x_i v_i\right)\left(\sum_{i=1}^3 y_i w_i\right) - \left(\sum_{i=1}^3 x_i w_i\right)\left(\sum_{i=1}^3 y_i v_i\right)$

$(X\times Y)(p)\cdot(v\times w) = (x_2y_3 - x_3y_2,\ x_3y_1 - x_1y_3,\ x_1y_2 - x_2y_1)\cdot(v_2w_3 - v_3w_2,\ v_3w_1 - v_1w_3,\ v_1w_2 - v_2w_1)$

$= x_2v_2y_3w_3 + x_3v_3y_2w_2 + x_3v_3y_1w_1 + x_1v_1y_3w_3 + x_1v_1y_2w_2 + x_2v_2y_1w_1$

$- x_2y_3v_3w_2 - x_3y_2v_2w_3 - x_3y_1v_1w_3 - x_1y_3v_3w_1 - x_1y_2v_2w_1 - x_2y_1v_1w_2$

$= \left(\sum_{i=1}^3 x_i v_i\right)\left(\sum_{i=1}^3 y_i w_i\right) - \left(\sum_{i=1}^3 x_i w_i\right)\left(\sum_{i=1}^3 y_i v_i\right)$

So $(W_X\wedge W_Y)(v,w) = (X\times Y)(p)\cdot(v\times w)$

18.1 $\varphi(t,\theta) = (t,\ y(t)\cos\theta,\ y(t)\sin\theta)$. $t\in I$, $\theta\in[0,2\pi)$

$E_1^\varphi = \frac{\partial\varphi}{\partial t} = (1,\ y'\cos\theta,\ y'\sin\theta)$, $E_2^\varphi = \frac{\partial\varphi}{\partial\theta} = (0,\ -y\sin\theta,\ y\cos\theta)$

$N = E_1^\varphi\times E_2^\varphi / \|E_1^\varphi\times E_2^\varphi\| = (1+y'^2)^{-1/2}(y',\ -\cos\theta,\ -\sin\theta)$

$L_p(E_1(p)) = -\frac{\partial N}{\partial t} = \left(\frac{-y''}{(1+y'^2)^{3/2}},\ \frac{-y''y'\cos\theta}{(1+y'^2)^{3/2}},\ \frac{-y''y'\sin\theta}{(1+y'^2)^{3/2}}\right) = -y''(1+y'^2)^{-3/2}(1,\ y'\cos\theta,\ y'\sin\theta)$

$L_p(E_2(p)) = -\frac{\partial N}{\partial\theta} = \left(0,\ \frac{-\sin\theta}{(1+y'^2)^{1/2}},\ \frac{\cos\theta}{(1+y'^2)^{1/2}}\right) = -\frac{1}{y(1+y'^2)^{1/2}}(0,\ -y\sin\theta,\ y\cos\theta)$

So $k_1(t,\theta) = -y''(t)/(1+y'^2)^{3/2}$, $k_2(t,\theta) = -\left(y(1+y'^2)^{\frac12}\right)^{-1}$

18.2 $E_1 = \frac{\partial\varphi}{\partial t} = (\cos\theta,\ \sin\theta,\ 0)$, $E_2 = \frac{\partial\varphi}{\partial\theta} = (-t\sin\theta,\ t\cos\theta,\ 1)$

$N = \frac{1}{\sqrt{1+t^2}}(\sin\theta,\ -\cos\theta,\ t)$. $L_p(E_1) = -\frac{\partial N}{\partial t} = \frac{-1}{(1+t^2)^{3/2}}(-t\sin\theta,\ t\cos\theta,\ 1) = -(1+t^2)^{3/2} E_2$

$L_p(E_2) = -\frac{\partial N}{\partial\theta} = -(1+t^2)^{-1/2}(\cos\theta,\ \sin\theta,\ 0) = -(1+t^2)^{-1/2} E_1$

So the matrix of L_p wrt E_1, E_2 is $\begin{pmatrix} 0 & -(1+t^2)^{-3/2} \\ -(1+t^2)^{-1/2} & 0\end{pmatrix}$. $H=0$.

18.3 By Ex 10.1. Let $\alpha(t) = (x(t), y(t))$, $t\in I$ then $k\circ\alpha = (x'y'' - x''y')/(x'^2+y'^2)^{3/2}$

If $k\equiv 0$, then $x'y'' - y'x'' = 0$. Since α is regular, so either $x'\neq 0$ or $y'\neq 0$

Suppose $y'\neq 0$, in some subinterval of I, then $\left(\frac{x'}{y'}\right)' = 0$, $\frac{x'}{y'} = C_1$, $x - C_2 = C_1(y - C_3)$