So $X = C_1 Y + C_2$. Suppose $X' \neq 0$ in some subinterval of I, similarly $Y = C_1' X + C_2'$.
It is obvious that a line segment parallel to x_1-axis and a line segment parallel to x_2-axis do not fit together smoothly, so S is a segment of a straight line.

18.4. Suppose the two principal curvatures are k_1, k_2. Then minimal surface $\Rightarrow k_1 + k_2 = 0$
So $K = k_1 * k_2 \le 0$.

18.5 Suppose the Weingarten map L_p has two eigenvalues λ_1, λ_2 corresponding to two eigenvectors V_1, V_2 which are orthonormal. $\forall \hat{V} \in S_p$, $\exists \alpha_1, \alpha_2 \in \mathbb{R}$, s.t. $\hat{V} = \alpha_1 V_1 + \alpha_2 V_2$.
So $k(\hat{V}) = L_p(\hat{V}) \cdot \hat{V} = (\alpha_1 \lambda_1 V_1 + \alpha_2 \lambda_2 V_2)(\alpha_1 V_1 + \alpha_2 V_2) = \alpha_1^2 \lambda_1 + \alpha_2^2 \lambda_2$
As S is minimal surface, so $\lambda_1 + \lambda_2 = 0$, $\lambda_2 = -\lambda_1$, $k(\hat{V}) = \lambda_1(\alpha_1^2 - \alpha_2^2)$
Now Let $V = \frac{\sqrt{2}}{2}(V_1 + V_2)$, $W = \frac{\sqrt{2}}{2}(V_1 - V_2)$, then $V \cdot W = 0$.
$k(V) = \lambda_1\left(\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2\right) = 0$. $k(W) = \lambda_1\left(\left(\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{2}}{2}\right)^2\right) = 0$.

18.6 $\underbrace{dN_p(v)}_{\forall v \in S_p} = \nabla_v N\big|_p = -L_p(v)$. Suppose the principal curvatures are λ_1, λ_2 corresponding to principal curvature directions V_1, V_2. Since V_1, V_2 span S_p, so $\exists \alpha_1, \alpha_2 \in \mathbb{R}$, $V = \alpha_1 V_1 + \alpha_2 V_2$
$\|dN_p(v)\| = \|L_p(v)\| = \|L_p(\alpha_1 V_1 + \alpha_2 V_2)\| = \|\alpha_1 \lambda_1 V_1 + \alpha_2 \lambda_2 V_2\| = \sqrt{\alpha_1^2 \lambda_1^2 + \alpha_2^2 \lambda_2^2}$
As S is minimal surface, $\lambda_1 = -\lambda_2$, So $\|dN_p(v)\| = |\lambda_1| \sqrt{\alpha_1^2 + \alpha_2^2} = |\lambda_1| \cdot \|V\|$.

18.7 $\cancel{\frac{d}{ds}\Big|_0 \ell(\alpha_s) = \frac{d}{ds}\Big|_0 \int_a^b \|\dot{\alpha}(s)\| dt = \int_a^b \frac{d}{ds} \|\dot{\alpha}(s)\|}$
$\frac{d}{ds}\Big|_0 \ell(\alpha_s) = \frac{d}{ds}\Big|_0 \int_a^b \|\dot{\alpha}_s(t)\| dt = \int_a^b \frac{d}{ds}\Big|_0 \|\dot{\alpha}_s(t)\| dt$ $(*)$
$\frac{d}{ds}\Big|_0 \|\dot{\alpha}_s(t)\| = \frac{d}{ds}\Big|_0 \sqrt{\dot{\alpha}_s(t) \cdot \dot{\alpha}_s(t)} = 2 \dot{\alpha}_s(t) \frac{d}{ds} \dot{\alpha}_s(t) / 2\|\dot{\alpha}_s(t)\| \Big|_{s=0}$
As $\|\dot{\alpha}_s(t)|_{s=0}\| = \|\dot{\alpha}(t)\| = 1$, $\dot{\alpha}_s(t)\big|_{s=0} = \dot{\alpha}(t)$
$\frac{d}{ds}\Big|_0 \dot{\alpha}_s(t) = \frac{\partial^2 \Psi(t,0)}{\partial s \partial t}$. So $\frac{d}{ds}\Big|_0 \|\dot{\alpha}(t)\| = \dot{\alpha}(t) \cdot \frac{\partial^2 \Psi(t,0)}{\partial s \partial t}$. Plugging into $(*)$
$\frac{d}{ds}\Big|_0 \ell(\alpha_s) = \int_a^b \dot{\alpha}(t) \cdot \frac{\partial^2 \Psi(t,0)}{\partial s \partial t} dt = \int_a^b \dot{\alpha}(t)\, d\frac{\partial \Psi(t,0)}{\partial s} = \int_a^b \dot{\alpha}(t)\, dX(t)$
$= \dot{\alpha}(t) X(t)\Big|_a^b - \int_a^b \ddot{\alpha}(t) X(t) dt$ (Note $X(t) = \frac{\partial \Psi(t,s)}{\partial s}\Big|_{s=0} := \frac{\partial \Psi(t,0)}{\partial s}$)
Using Ex 10.6, $\ddot{\alpha}(t) = k(t) N$, we have $\frac{d}{ds}\Big|_0 \ell(\alpha_s) = \dot{\alpha}(t) X(t)\Big|_a^b - \int_a^b (X \cdot N) k(t) dt$
If $\Psi(a,s) = \alpha(a)$, $\Psi(b,s) = \alpha(b)$ $\forall s$ (i.e., compactly supported), then
$X(a) = 0 = X(b)$ So $\frac{d}{ds}\Big|_0 \ell(\alpha_s) = -\int_a^b (X \cdot N) k(t) dt$.