~~(b)~~ if $g(t_1)<0$, $g(t_2)<0$, same contradiction occurs. So $g(t_1)g(t_2)<0$

(b) If α crosses S for an odd number of times $t_1 \cdots t_n$ then by (a) $g'(t_1)g'(t_n)>0$, With out loss of generality, suppose $g'(t_1)>0$, $g'(t_n)>0$.

Since $g(t_1)=g(t_n)=c$ and t_1, t_n are two extreme times, So $g(t)<c$ for all $t<t_1$; $g(t)>c$ for all $t>t_n$

However as S is compact ~~and α goes to ∞ in both directions~~ ~~we can~~ and $f: R^{n+1} \to R$ ~~there is a bar~~ Suppose S is strictly contained in sphere $S': \|x\|^2=r^2$. ~~then pick any $p \in S$ and consider $S_1 f^{-1}(f(p))$~~. Since α goes to ∞ in both directions

There must be t_0, t_{n+1} with $t_0<t_1$, $t_{n+1}>t_n$, such that $\alpha(t_0)$ and $\alpha(t_{n+1}) \in S'$

~~As $f(\alpha(t_0))<c$, $f(\alpha(t_{n+1}))>c$ and f is continuous on S'~~, So $f(\alpha(t_0))<c$, $f(\alpha(t_{n+1}))>c$

As S' is connected (see Ex. 5.1), there is a continuous parametrized curve $\beta(t) \in S'$, s.t. $\beta(t^1)=\alpha(t_0)$, $\beta(t^2)=\alpha(t_{n+1})$, As f, β are continuous on S', there must be a $t^3 \in (t^1, t^2)$ s.t. $f(\beta(t^3))=c$.

But $\beta(t^3) \in S'$, So $\beta(t^3) \bar{\in} S$. This is contradiction!

6.10 (a) ~~$\forall \beta(t)$~~. Since $\beta(a) \in O(S)$, there exists a continuous map $\alpha: [0,+\infty) \to R^{n+1}-S$ s.t. $\alpha(0)=\beta(a)$, $\lim_{t\to\infty}\|\alpha(t)\|=\infty$.

For $\forall \beta(t_0)$. Construct curve $\gamma(t) := \begin{cases} \beta(t_0-t) & t\in[0, t_0-a] \\ \alpha(t-t_0+a) & t \in (t_0-a,+\infty) \end{cases}$

then $\gamma(t)$ is continuous from $[0,+\infty) \to R^{n+1}-S$. $\gamma(0)=\beta(t_0)$, $\gamma(+\infty)=\alpha(+\infty)=\infty$

t_0 is arbitrary so $\beta(t) \in O(S)$ for all $t \in [a,b]$

(b) ~~Open set. $\forall p$~~ (1) Non-empty. As S is a compact n-surface, ~~there~~ we can find a n-sphere with a large enough radius r, which strictly subsumes S then pick one point on the n-sphere, p, construct continuous map $\alpha(t)=pt+p$. So $\alpha(0)=p$. $\forall t>0$. $\|\alpha(t)\|=(t+1)r>r$

So $\alpha(t) \in R^{n+1}-S$, $\lim_{t\to\infty}\|\alpha(t)\|=\infty$. So $p \in O(S)$

(2) open set $\forall p \in O(S)$, ~~Ran~~ $p \in R^{n+1}-S$, as $R^{n+1}-S$ is open (due to $S=f^{-1}(c)$ is n-surface and by definition f is smooth). So there exists an ε-ball around p, (p,ε), such that $\forall x \in (p,\varepsilon)$ satisfy $x \in R^{n+1}-S$ we can easily construct a continuous map from p to x, By (a), $x \in O(S)$, $\forall x \in (p,\varepsilon)$

(3) connected: $\forall p, q \in O(S)$, Suppose there is a n-sphere S_1 with radius r ~~s.t.~~ such that p, q, S are all contained in it (S compact, $r>\|p\|$, $r>\|q\|$).

As $p \in O(S)$ there is a continuous map $\alpha_1: [0,+\infty) \to R^{n+1}-S$, $\alpha_1(0)=p$, $\lim_{t\to\infty}\|\alpha_1(t)\|=\infty$.

Suppose $\|\alpha_1(t_1)\|=r$ (i.e. $\alpha_1(t_1) \in S_1$), Likewise, we define $\alpha_2(t)$ and t_2

As S_1 is connected and $S_1 \subset R^{n+1}-S$, there's a curve α_3 on S_1, s.t. $\alpha_3(a)=\alpha_1(t_1)$,